# Unit 5 - Coordinate Methods

## Lesson 2 PRACTICE PROBLEMS
Coordinate Models of Transformations

**I can use coordinates to model transformations and investigate their properties.**

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<th>Investigation</th>
<th>Practice Problem Options</th>
<th>Max Possible Points</th>
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<tr>
<td>Investigation 1: Modeling Rigid</td>
<td>#1, 2, 3, 4, 5, 6, 7</td>
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<td>Transformations</td>
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<td>Investigation 2: Modeling Size</td>
<td>#8, 9, 10</td>
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<td>Investigation 3: Combining Transformations</td>
<td>#11, 12, 13, 14</td>
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**In order to earn credit for practice problems, ALL WORK must be shown.**

_________/40 points
INVESTIGATION 1

1. Copy each polygon below on a separate coordinate grid. Draw and label the transformed image according to the given rule. Identify as precisely as you can the type of transformation.

   a. \((x, y) \rightarrow (x + 2, y - 3)\)
   b. \((x, y) \rightarrow (-y, x)\)
   c. \((x, y) \rightarrow (-x, y)\)
   d. \((x, y) \rightarrow (-x, -y)\)

2. \(\triangle ABC\) has vertices as follows: \(A(1, 2), B(4, 4),\) and \(C(3, 6)\).
   a. Draw \(\triangle ABC\) on a coordinate grid. Then draw and label the image of \(\triangle ABC\) under each of the following transformations.
      i. Translation with horizontal component 5 and vertical component -4
      ii. Horizontal translation 7 units to the left
      iii. Translation that maps the origin to the point \((-3, -6)\)
   b. Choose one of the image triangles in Part a and verify that it is congruent to \(\triangle ABC\).
3 \( \triangle PQR \) has vertices as follows: \( P(3, -2) \), \( Q(6, -1) \), and \( R(4, 3) \).

a. On separate coordinate grids, draw \( \triangle PQR \) and its image under each of the following transformations. Label the vertices of the images.

i. Rotation of 180° about the origin

ii. Rotation of 90° counterclockwise about the origin

iii. Rotation of 90° clockwise about the origin

b. Choose one of the image triangles in Part a and verify that it is congruent to \( \triangle PQR \).

4 Consider \( \square ABCD = \begin{pmatrix} 1 & 2 & 6 & 5 \\ -1 & 2 & 2 & -1 \end{pmatrix} \).

a. On separate coordinate grids, draw \( \square ABCD \) and its image under each of the following transformations. Label the vertices of the images.

i. Reflection across the \( x \)-axis

ii. Reflection across the line \( y = x \)

iii. Reflection across the \( y \)-axis

b. Choose one of the image quadrilaterals in Part a and verify that it is a parallelogram.

c. What is the perimeter of \( \square ABCD \)? How do you know that each of the image parallelograms in Part a will have the same perimeter?

5 Consider \( \triangle PQR = \begin{pmatrix} -2 & 2 & 0 \\ -1 & 1 & 5 \end{pmatrix} \).

a. What kind of triangle is \( \triangle PQR \)? How do you know?

b. What is the area of \( \triangle PQR \)?

c. On separate coordinate grids, draw \( \triangle PQR \) and its image under each of the following transformations. Label the vertices of the images.

i. Translation that maps the origin to the point \((-2, -2)\)

ii. Counterclockwise rotation of 270° about the origin

iii. Reflection across the line \( y = -x \)

d. What kind of triangle is each of the three image triangles in Part c? How do you know?

e. Find and compare the areas of the three image triangles in Part c.
A triangle translation program that implements the translation planning algorithm in Investigation 1, Problem 5 (page 199) is given below.

**a. Analyze this program and explain the purpose of each command line not already described.**

<table>
<thead>
<tr>
<th>TRANSNL Program</th>
<th>Function in Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>ClrHome</td>
<td>1. Clears the home screen.</td>
</tr>
<tr>
<td>Input “X1 COORD-PRE”,A</td>
<td>2. Requests input for x-coordinate of one vertex. Stores the value in variable named A.</td>
</tr>
<tr>
<td>Input “Y1 COORD-PRE”,B</td>
<td>3. Requests input for y-coordinate of one vertex.</td>
</tr>
<tr>
<td>Input “Y2 COORD-PRE”,D</td>
<td>5. Requests input for y-coordinate of another vertex.</td>
</tr>
<tr>
<td>Input “Y3 COORD-PRE”,F</td>
<td>7. Requests input for y-coordinate of the third vertex.</td>
</tr>
<tr>
<td>Input “X COMP-TRANS”,H</td>
<td>8. Requests input for the horizontal component of the translation. Stores the value in variable $H$.</td>
</tr>
<tr>
<td>ClrHome</td>
<td>10. Clears the home screen.</td>
</tr>
<tr>
<td>Disp “PREIMAGE”</td>
<td>11. Displays “PREIMAGE”.</td>
</tr>
<tr>
<td>Pause</td>
<td>12. Pauses the program execution until Enter is pressed.</td>
</tr>
<tr>
<td>ClrDraw</td>
<td>13. Clears all drawings.</td>
</tr>
<tr>
<td>Line(A,B,C,D)</td>
<td>14. Draws a line segment from vertex $(A, B)$ to vertex $(C, D)$.</td>
</tr>
<tr>
<td>Line(C,D,E,F)</td>
<td>15. Draws a line segment.</td>
</tr>
<tr>
<td>Line(E,F,A,B)</td>
<td>16. Draws a line segment.</td>
</tr>
<tr>
<td>Pause</td>
<td>17. Pauses the program execution.</td>
</tr>
<tr>
<td>ClrHome</td>
<td>18. Clears the home screen.</td>
</tr>
<tr>
<td>Disp “IMAGE”</td>
<td>19. Displays “IMAGE”.</td>
</tr>
<tr>
<td>Pause</td>
<td>20. Pauses the program execution.</td>
</tr>
</tbody>
</table>
Look back at the TRANSL program in Applications Task 6.

**a.** How would you modify the program to display a triangle and its image under a 90° counterclockwise rotation about the origin?

**b.** Enter the modified program, name it ROT90 in your calculator, and test the program with a sample triangle.

8. Copy each polygon below on a separate coordinate grid. Draw and label the transformed image according to the given rule. Identify as precisely as you can the type of transformation.

- **a.** \((x, y) \rightarrow (3x, 3y)\)
- **b.** \((x, y) \rightarrow (-x + 8, y)\)
- **c.** \((x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)\)
- **d.** \((x, y) \rightarrow (x, -y - 4)\)

9. A picture is to be placed in a brochure, and the designer wants it positioned in a 2" \(\times\) 3" frame. If the original picture is 6" \(\times\) 9", what size transformation should be applied to the picture so it will fit the frame?
Consider quadrilateral \(ABCD = \begin{bmatrix} 6 & -4 & -3 & 7 \\ 2 & 0 & -5 & -3 \end{bmatrix}\).

a. Draw quadrilateral \(ABCD\) on a coordinate grid.

b. Draw the image quadrilateral, \(A'B'C'D'\), resulting from transforming \(ABCD\) with a size transformation of magnitude 2.5 and center at the origin.

c. How do \(\overline{AB}\) and \(\overline{BC}\) appear to be related? How do \(\overline{A'B'}\) and \(\overline{B'C'}\) appear to be related? Verify your conjectures using coordinates.

d. How do \(\overline{AB}\) and \(\overline{A'B'}\) appear to be related? How do \(\overline{BC}\) and \(\overline{B'C'}\) appear to be related? Verify your conjectures using coordinates.

e. Find the area of quadrilateral \(ABCD\).
   i. Predict the area of quadrilateral \(A'B'C'D'\).
   ii. Check your prediction.

f. Connect each preimage point and its image with a line. What is true about the lines?

Preimage and image pairs of a figure under certain transformations are shown below. The image figure is darker blue. In each case, identify as precisely as you can the type of transformation. Then write a coordinate rule for the transformation.
Size transformations have their center at the origin. However, in producing graphics displays, it is sometimes useful to enlarge or reduce a figure using a center different from the origin.

a. Consider the following procedure for applying a size transformation to a figure when the center of the size transformation is \(A(2, 1)\) and the magnitude is 3.

Draw \(\triangle PQR\) on a coordinate grid. Plot point \(A\).

\[
\triangle PQR = \begin{bmatrix} 3 & 4 & 4 \\ 3 & 3 & 5 \end{bmatrix}
\]

Step 1: Determine the horizontal and vertical components of the translation that will translate \(A(2, 1)\) to the origin. Find the image of \(\triangle PQR\) under that translation. Label as \(\triangle 1\) the image of \(\triangle PQR\).

Step 2: Apply a size transformation to \(\triangle 1\) using the origin as center and 3 as the magnitude. Label the new image \(\triangle 2\).

Step 3: Find the components of the translation that maps the origin back to \(A(2, 1)\). Then find the image of \(\triangle 2\) under that transformation. Label the final image \(\triangle P'Q'R'\).

b. Examine \(\triangle PQR\) and \(\triangle P'Q'R'\). Does this procedure produce the desired result—that is, a size transformation of magnitude 3 with center \(A(2, 1)\)? Explain.

c. Write a symbolic rule \((x, y) \rightarrow (__, __)\) that describes this composite transformation.

Modify the “translate-transform-translate back” procedure outlined in Applications Task 12 to create a procedure for rotating \(\triangle PQR\) 90° counterclockwise about the point \(A(2, 1)\). Then write a symbolic rule \((x, y) \rightarrow (__, __)\) that describes this composite transformation.

For each of the following pairs of triangles, describe a composition of two or more transformations that will map the first triangle to the second triangle. Write a symbolic rule \((x, y) \rightarrow (__, __)\) that describes the composite transformation.

a. \(\triangle I\) onto \(\triangle II\)
b. \(\triangle I\) onto \(\triangle III\)
c. \(\triangle II\) onto \(\triangle IV\)
d. \(\triangle I\) onto \(\triangle IV\)